

AD-A145 555

APPLICATIONS OF FUNCTIONAL ANALYTIC AND MARTINGALE  
METHODS(U) MASSACHUSETTS UNIV AMHERST DEPT OF  
MATHEMATICS AND STATISTICS W A ROSENKRANTZ JUL 84

1/1

UNCLASSIFIED

AFOSR-TR-84-0718 AFOSR-82-0167

F/G 12/2

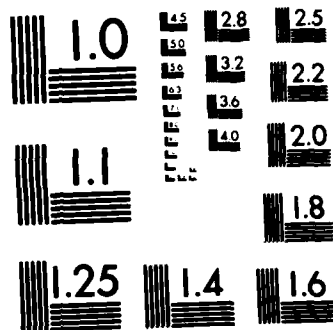
NL



END

FILMED

DTIC



MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS-1963-A

AFOSR-TR. 8218

(4)

APPLICATIONS OF FUNCTIONAL ANALYTIC AND MARTINGAL METHODS

ANNUAL SCIENTIFIC REPORT

GRANT AFOSR-82-0167

15 MAY 1983 - 14 MAY 1984

AD-A145 555

Walter A. Rosenkrantz, Department of Mathematics & Statistics  
University of Massachusetts, Amherst MA 01003

DTIC FILE COPY

SEP 10 1984  
A

Approved for publication  
distribution

84 08 30 035

Title: Applications of Functional Analytic and Martingale Methods --  
Annual Scientific Report on AFOSR Grant 82-0167

Principal Investigator: Professor Walter A. Rosenkrantz

I. Publications

1. Calculation of the Laplace transform of the length of the busy period for the M/G/1 Queue via Martingales, Annals of Probability, Vol. 11, No. 3, pp 817-818, August 1983.
2. Diffusion Approximation for a class of Markov processes satisfying a nonlinear Fokker-Planck equation (with Li Zhan Bing of Beijing Normal University), Journal of Nonlinear analysis, Vol. 7, No. 10, pp 1089-1099, 1983.
3. On the Instability of the Slotted ALOHA Multiaccess Algorithm (with D. Towsley), IEEE Transactions on Automatic Control, Vol. AC-28, No. 10, pp 994-996, October 1983.
4. Weak Convergence of a Sequence of Queueing and Storage processes to a Singular Diffusion, AFOSR 82-0167, Tech Report No. 4; presented to "International Seminar on Modelling and Performance Evaluation Methodology" held in Paris, France, January 24-26, 1983. To appear in a forthcoming volume in the "Lecture Notes in Control and Information Sciences".
5. Some Theorems on the Instability of the Exponential Back-off Protocol, AFOSR 82-0167, Report No. 7, January 1984. This paper is currently being refereed.

II. In preparation.

- (1) Weak convergence of a sequence of Markov processes to a multivariate Ornstein-Uhlenbeck process (joint with F. Bennett)

III Coupling activities i.e. Lectures, Conferences, Symposia, etc.

- (1) Presented a lecture to the Performance Analysis Group at the T.J. Watson Research Labs of IBM in Yorktown Heights in June 1983. I met and consulted with Dr's. Philip Heidelberger, Randy Nelson, and D. Towsley.
- (2) I attended the AMS summer seminar on Stochastic Differential Equations and Applications held in Boulder, Colorado, July 25-29, 1983 and presented my paper "Some Theorems on the Instability of the Exponential Back-off Protocol" (AFOSR 82-0167, Report No. 7) in the special session on "Infinite Interacting particle systems" chaired by R. Durrett.
- (3) I attended the 22nd IEEE conference on Decision and Control held in San Antonio, Texas, Dec. 14-16, 1983 and attended the session on Stochastic Control.
- (4) The principal investigator invited Dr.'s Ruth Williams (Courant Institute), Zhong Xin Zhao (Institute of Systems Science and Mathematical Sciences,

Academia Sinica, Beijing, China), Julian Besag, L. Snell, (Dartmouth College), Phil Heidelberger (IBM, Yorktown Heights) to consult with him on mathematical problems that arose in the course of his research.

#### IV. Professional Personnel associated with research effort.

- (1) F. Bennett completed her Ph.D. in August 1983 with a thesis written under my direction entitled "On a sequence of Markov Processes converging to a Multivariate Ornstein-Uhlenbeck Process". A revised version of this thesis is currently in preparation for publication. The original version appeared as AFOSR 82-0167 Report No. 6.
- (2) Bill Link was supported last summer in his research on a Martingale approach to the statistical analysis of censored data. He is currently continuing his research with my colleague Professor R. Kowar.


#### V. Status of Current Research.

The goal of our research is mathematical modeling and analysis of various random access protocols that have been proposed for computer communication networks. Of particular concern is the stability and throughput of the protocol, where instability means the backlog of packets awaiting to be retransmitted tends to infinity with probability one i.e. the system crashes. For example we have discovered that the "exponential backoff protocol" used in the Ethernet system can be modeled as an infinite particle system with non local interactions. For this protocol the backlog at time is no longer a Markov process and new methods had to be developed in order to study its stability properties.

Our contribution to this problem has been to develop computational methods based on Martingale theory for studying the time dependent behavior of the expected backlog. In the case of the "exponential backoff protocol" we showed that when the arrival rate  $\lambda$  exceeds a certain critical value  $\lambda_c$  the backlog is a non negative submartingale which yields a lower bound on the expected backlog. An immediate consequence of this estimate is that the "exponential backoff protocol", at least in the infinite user case, is unstable. We are currently investigating the stability properties of the protocol when the arrival rate  $\lambda \leq \lambda_c$ . We plan to supplement our theoretical studies with computer simulations. It is worth pointing out that our Martingale methods have led to a new result even in the case when the backlog (as in the case of the slotted ALOHA algorithm) is a Markov process.

Long range goals include: (i) extensions of the Martingale methods mentioned above to the study of the time dependent behavior of higher order moments of the stochastic process which models the multi user communications network; (ii) modeling and analysis of "fault tolerant computing systems" which is closely linked to the so-called "non-stop operating systems." We are particularly interested in the mean response time of such a system; (iii) the behavior of the protocol when the arrival rate approaches the critical value i.e. we plan to use the diffusion approximation in its modern form due to Trotter, Kurtz, Stroock and Varadhan. In some cases of interest it has been shown that the limiting diffusion process is no longer a Brownian motion process but is instead a Bessel process. In the case of two or more dimensions the limiting process is Brownian motion in a quadrant with an oblique reflection at the boundary.

# REPORT DOCUMENTATION PAGE

1. REPORT SECURITY CLASSIFICATION UNCLASSIFIED			2. RESTRICTIVE MARKINGS		
3. SECURITY CLASSIFICATION AUTHORITY			4. DISTRIBUTION AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.		
5. DECLASSIFICATION/DOWNGRADING SCHEDULE			6. PERFORMING ORGANIZATION REPORT NUMBER(S)		
7. NAME OF PERFORMING ORGANIZATION University of Massachusetts			8. OFFICE SYMBOL (If applicable)		
9. ADDRESS (City, State, and ZIP Code) Dept of Mathematics & Statistics Amherst MA 01003			10. NAME OF MONITORING ORGANIZATION Air Force Office of Scientific Research Directorate of Mathematical & Information Sciences, AFOSR, Bolling AFB DC 20332		
11. NAME OF FUNDING/SPONSORING ORGANIZATION AFOSR			12. OFFICE SYMBOL (If applicable) NA		
13. ADDRESS (City, State, and ZIP Code) Bolling AFB DC 20332			14. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER AFOSR-82-0167		
15. TITLE (Include Security Classification) APPLICATIONS OF FUNCTIONAL ANALYTIC AND MARTINGAL METHODS			16. SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO 61102F		
			PROJECT NO 1304		
			TASK NO A5		
			WORK UNIT ACCESSION NO		
17. PERSONAL AUTHOR(S) Walter A. Rosenkrantz					
18a. TYPE OF REPORT Interim		18b. TIME COVERED FROM 15/5/83 TO 14/5/84		19. DATE OF REPORT (Year, Month, Day) JUL 84	
20. PAGE COUNT 2					
21. SUPPLEMENTARY NOTATION					
22. COSATI CODES			23. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP			
24. ABSTRACT (Continue on reverse if necessary and identify by block number)					
<p>This annual report surveys and summarizes the principal investigator's current research on modeling and analysis of various random access protocols via Martingale and functional analytical methods.</p>					
<div style="text-align: center;">  </div>					
25. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			26. ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
27. NAME OF RESPONSIBLE INDIVIDUAL CPT Brian W. Woodruff			28. TELEPHONE (Include Area Code) 1-202-767-5027		29. OFFICE SYMBOL NA

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted

All other editions are obsolete

SECURITY CLASSIFICATION OF THIS PAGE

84 08 30 035